fIRTree: An Item Response Theory modeling of fuzzy rating data

Antonio Calcagnì

DPSS, University of Padova

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As they involve human raters, rating data are often affected by **fuzziness** because of the **decision uncertainty** that affects the **response process**.

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In this context, several methods can be used to quantify fuzziness (**fuzzy scaling**):

- direct fuzzy rating [5]
- implicit fuzzy rating [2]
- deterministic crisp-to-fuzzy conversion systems [8]
- statistically-oriented crisp-to-fuzzy conversion systems [9]

Besides their differences, all these approaches aim at quantifying the fuzziness present in rating data.

In this presentation, we will describe a new statistically-oriented crisp-to-fuzzy conversion method (i.e., **fIRTree**), which is based on a psychometric modeling of the rating process (IRTree).

The purpose is to provide a method which revolves around the modeling of the stage-wise cognitive steps used during the rating process.

More technical details and extended results are available in [3, 4].

IRTree

A psychometric modeling of the rating process

fIRTree is based on **IRTree** [1], a novel class of Item Response Theory models that formalizes the steps required by a rater to provide the rating response.

To describe how IRTree works, consider again the previous example:

- I am satisfied with my current work -

Strongly disagree — Disagree — 0 — Agree — Strongly agree

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Then, each **response option** is thought as being the output of a cognitive sub-process of the entire response process. The sub-processes are modeled as **nodes** of a **binary tree**.

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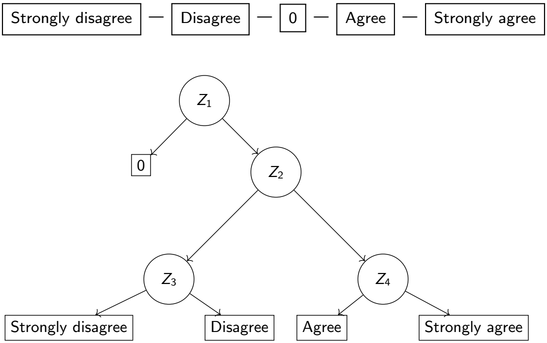
IRTree

A psychometric modeling of the rating process

An example of 5-point rating scale with the associated binary decision tree.



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IRTree

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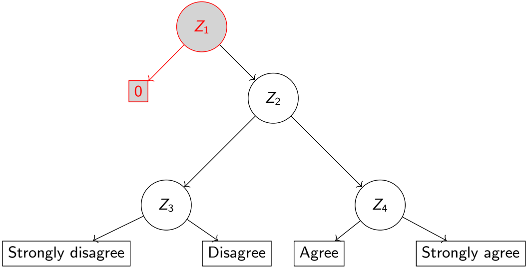
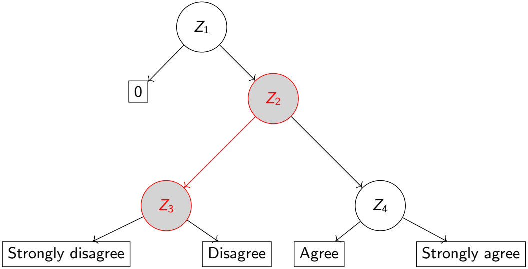
In this schema, the rater:

- first decides **whether or not** provide a response ($Z_1 \in \{0, 1\}$)



In this schema, the rater:

- first decides whether or not provide a response ($Z_1 \in \{0, 1\}$)
- then, for $Z_1 = 1$ he/she decides the **direction** of the response, if **negative** ($Z_2 = 0$)



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- first decides whether or not provide a response ($Z_1 \in \{0, 1\}$)
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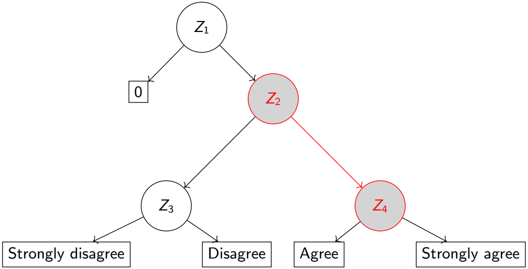

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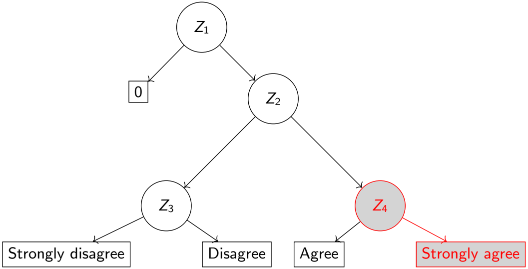
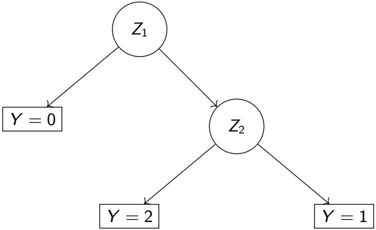
- first decides whether or not provide a response ($Z_1 \in \{0, 1\}$)
- then, for $Z_1 = 1$ he/she decides the **direction** of the response, if negative ($Z_2 = 0$) or **positive** ($Z_2 = 1$)
- finally, he/she decides the **strength** of the response, e.g. "Strongly agree" ($Z_4 = 1$)



IRTree

A psychometric modeling of the rating process

Depending on the rating model being adopted, several schemata can be adopted for this purpose such as:



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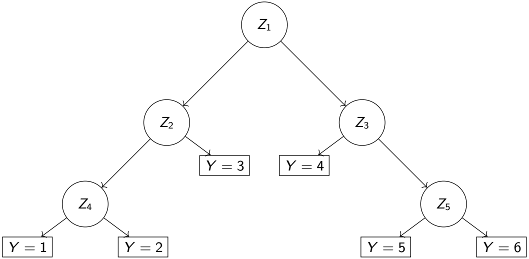

More formally, the IRTree model is defined by the following equations (i-th rater, j-th item/question, n-th node):

$$
\begin{aligned}
&Z_{ijn} \sim \mathcal{B}er(\pi_{ijn}) \\
&\pi_{ijn} = \mathbb{P}(Z_{in} = 1; \boldsymbol{\theta}_n) = \frac{\exp(\eta_{in} + \alpha_{jn})}{1 + \exp(\eta_{in} + \alpha_{jn})} \\
&\eta_{in} \sim \mathcal{N}(\mathbf{0}, \boldsymbol{\Sigma}_\eta)
\end{aligned}
$$

where

$\alpha_{jn} \in \mathbb{R}$: easiness of the **item** being rated

$\eta_{in} \in \mathbb{R}$: **rater**'s latent ability to answer the question

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where

$$\mathbb{P}(Y_i = m; \boldsymbol{\theta}_n) = \prod_{n=1}^{N} \mathbb{P}(Z_{in} = d; \boldsymbol{\theta}_n)^d$$

is the probability of the response $Y_i = m$ for the item being rated.

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The parameters $\boldsymbol{\theta}_n = \{\boldsymbol{\alpha}, \boldsymbol{\Sigma}_\eta\}$ can be estimated via **marginal maximum likelihood** [1].

Once $\hat{\boldsymbol{\alpha}}$ and $\widehat{\boldsymbol{\Sigma}}_\eta$ have been recovered conditioned on a sample of data $\mathbf{Y}_{I\times J}$, the estimated **transition probabilities**

$$\mathcal{U}_i = \Big(\hat{\mathbb{P}}(Y_i = 1), \ldots, \hat{\mathbb{P}}(Y_i = m), \ldots, \hat{\mathbb{P}}(Y_i = M)\Big)$$

provide information about the **decision uncertainty** of the rater's response process.

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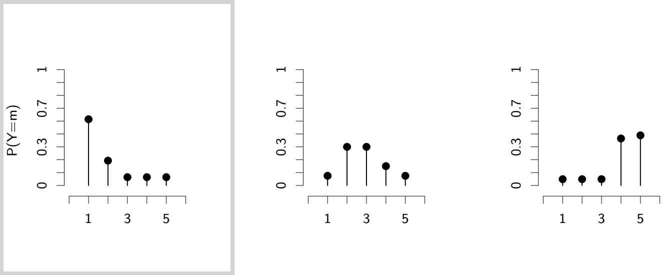

Response process with lower degree of decision uncertainty
(i.e., the response $Y = 1$ is more certain than the remaining ones).

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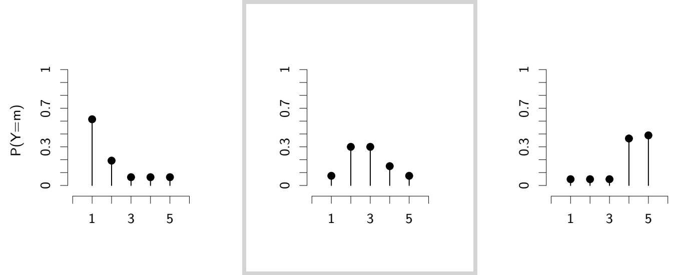
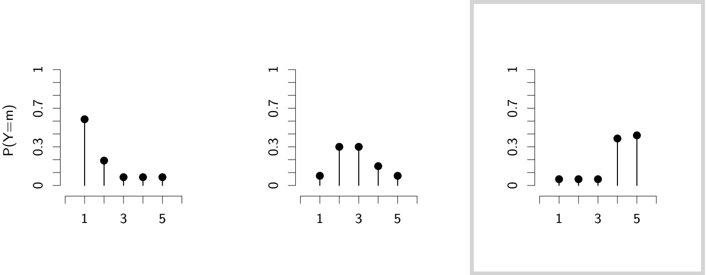
$$\mathcal{U}_i = \Big(\hat{\mathbb{P}}(Y_i = 1), \ldots, \hat{\mathbb{P}}(Y_i = m), \ldots, \hat{\mathbb{P}}(Y_i = M)\Big)$$



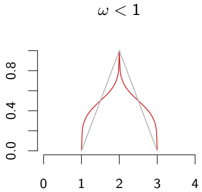
Response process with higher degree of decision uncertainty
(i.e., both $Y \in \{2, 3\}$ responses are probable).

$$\mathcal{U}_i = \left(\hat{\mathbb{P}}(Y_i = 1), \ldots, \hat{\mathbb{P}}(Y_i = m), \ldots, \hat{\mathbb{P}}(Y_i = M)\right)$$



Response process with a certain degree of decision uncertainty (i.e., both $Y \in \{4, 5\}$ responses are probable).

fIRTree quantifies the fuzziness of the rating process by means of **4-parameter triangular fuzzy sets** [6], where an additional parameter $\omega \in \mathbb{R}_0^+$ is used to **intensify** ($\omega < 1$) or **deflate** ($\omega > 1$) the fuzziness of the fuzzy set.



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The 4-parameter triangular fuzzy sets are built from the estimated transition probabilities $\mathcal{U}_i$ as follows:

- The **mode** of the set is equated to the **expected value** of $\mathcal{U}_i$
- The **left** and **right spreads** of the set are computed via transformations of the **variance** of $\mathcal{U}_i$ (i.e., using moment-matching equations)
- The parameter ω is computed as the **difficulty of responding** to the item/question: $\omega_{ij} = \sum_{m=1}^{M} \hat{\mathbb{P}}(Y_i = m)^2$

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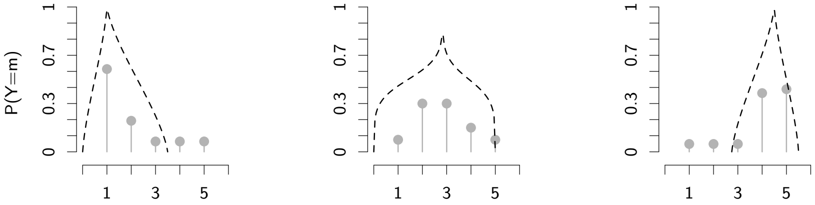

Note: The cases where $\omega < 1$ indicate that the rater has been hesitant in providing his/her final response.

A **simulation study** was run to evaluate the performance of fIRTree in recovering decision uncertainty from rating data.

To this purpose, a controlled scenario based on *simulated faking data* was used to control the amount of decision uncertainty in the simulated scenario. Faking behaviors in rating situations can serve as a way to study the levels of decision uncertainty in rating data [7].

Simulation study

Design and procedure

- **Design**
  Three factors $I \in \{50, 100, 150\}$ (sample size), $J \in \{10, 20\}$ (number of items/questions), $\xi \in \{0, 0.25, 0.50, 0.75\}$ (degree of faking in the data) were varied in a complete factorial design with $B = 1000$ samples.

  The number of response categories were held fixed ($M = 5$, i.e.: 5-point rating scale). The simplest IRTree schema with $N = 4$ nodes was used (see slide 5).

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Simulation study
Design and procedure

- **Data generation**
  1. Rating data (crisp) were generated using the IRTree model
  2. The SGR faking method [7] was used to perturb crisp data according to an increasing pattern of decision uncertainty
  3. Fuzzy numbers were computed using fIRTree

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Simulation study

Design and procedure

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- **Outcome measures**
  Fuzziness of the fuzzy sets as computed by the Kauffmann index (the higher the index, the largest the fuzziness of the set).

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Simulation study

A sketch of the results

		$\xi = 0$ (baseline)	$\xi = 0.25$ (low DU)	$\xi = 0.5$ (medium DU)	$\xi = 0.75$ (high DU)
	$I = 50$	0.617 (0.075)	0.724 (0.041)	0.784 (0.019)	0.815 (0.01)
$J = 10$	$I = 150$	0.602 (0.058)	0.716 (0.026)	0.779 (0.01)	0.812 (0.006)
	$I = 500$	0.603 (0.062)	0.717 (0.028)	0.78 (0.008)	0.813 (0.004)
	$I = 50$	0.613 (0.079)	0.72 (0.045)	0.781 (0.022)	0.814 (0.01)
$J = 20$	$I = 150$	0.599 (0.062)	0.713 (0.03)	0.776 (0.011)	0.811 (0.006)
	$I = 500$	0.6 (0.066)	0.714 (0.032)	0.777 (0.011)	0.812 (0.004)

Kauffmann index as a function of the degrees of decision uncertainty (DU). Note that the faking factor ξ is order from lower to higher DU.

Main results

The fuzziness of the fIRTree-based fuzzy sets increased as decision uncertainty increased regardless of sample size I and number of items J.

Conclusions

fIRTree:

- It is a method used to quantify fuzziness from crisp rating data
- It is based on a psychometric modeling of the rating process (IRTree)
- It can be easily used in many applicative contexts involving human rating data (e.g., see [3, 4] for some case studies)

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antonio.calcagni@unipd.it

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